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On Bisectors for Convex Distance Functions

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Abstract: It is well known that the construction of *Voronoi diagrams* is based on the notion of *bisector* of two given points. Already in normed linear spaces, bisectors have a complicated structure and can, for many classes of norms, only be described with the help of topological methods. Even more general, we present results on bisectors for convex distance functions (gauges). Let C, with the origin o from its interior, be the compact, convex set inducing a *convex distance function* (gauge) in the plane, and let $B(-x,x)$ be the *bisector* of $-x$ and x, i.e., the set of points z whose distance (measured with the convex distance function induced by C) to $-x$ equals that to x. For example, we prove the following characterization of the Euclidean norm within the family of all convex distance functions: if the set L of points x in the boundary ∂C of C that create $B(-x,x)$ as a straight line has non-empty interior with respect to ∂C, then C is an ellipse centered at the origin. For the subcase of normed planes we give an easier approach, extending the result also to higher dimensions.

Key words: Birkhoff orthogonality, bisector, characterization of ellipse, convex distance function, Euclidean norm, gauge, isosceles orthogonality, Roberts orthogonality, Voronoi diagram.

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References

[1] A. Aizpuru, F.J. García-Pacheco, F. Rambla, Isometric reflection vectors in Banach spaces, *J. Math. Anal. Appl.* **299** (1) (2004), 40 – 48.

[2] J. Alonso, C. Benítez, Orthogonality in normed linear spaces: a survey, I, Main properties, *Extracta Math.* **3** (1) (1988), 1 – 15.

[3] J. Alonso, H. Martini, S. Wu, On Birkhoff orthogonality and isosceles orthogonality in normed linear spaces, *Aequationes Math.* **83** (1-2) (2012), 153 – 189.

[4] D. Amir, "Characterizations of Inner Product Spaces", Operator Theory: Advances and Applications, 20, Birkhäuser Verlag, Basel, 1986.

[5] F. Aurenhammer, Voronoi diagrams–a survey of a fundamental geometric data structure, *ACM Comput. Surv.* **23** (3) (1991), 345 – 405.

[6] J.B. Guerrero, A.R. Palacios, Isometric reflections on Banach spaces after a paper of A. Skorik and M. Zaidenberg: "On isometric reflections in Banach spaces" [*Mat. Fiz. Anal. Geom.* **4** (1-2) (1997), 212 – 247], *Rocky Mountain J. Math.* **30** (1) (2000), 63 – 83.

[7] J.-B. Hiriart-Urruty, C. Lemaréchal, "Fundamentals of Convex Analysis", Grundlehren Text Editions, Springer-Verlag, Berlin, 2001.

[8] Á.G. Horváth, On bisectors in Minkowski normed spaces, *Acta Math. Hungar.* **89** (3) (2000), 233 – 246.

[9] Á.G. Horváth, Bisectors in Minkowski 3-space, *Beiträge Algebra Geom.* **45** (1) (2004), 225 – 238.

[10] R.C. James, Orthogonality in normed linear spaces, *Duke Math. J.* **12** (1945), 291 – 302.

[11] R.C. James, Orthogonality and linear functionals in normed linear spaces, *Trans. Amer. Math. Soc.* **61** (1947), 265 – 292.

[12] R. Klein, "Concrete and Abstract Voronoi Diagrams", Lecture Notes in Computer Science, Springer-Verlag, Berlin, 1989.

[13] Lihong Ma, "Bisectors and Voronoi Diagrams for Convex Distance Functions", PhD thesis, Fern-Universität Hagen, 2000.

[14] H. Martini, K.J. Swanepoel, The geometry of Minkowski spaces–a survey, II, *Expo. Math.* **22** (2) (2004), 93 – 144.

[15] H. Martini, K.J. Swanepoel, G. Weiss, The geometry of Minkowski spaces–a survey, I, *Expo. Math.* **19** (2) (2001), 97 – 142.

[16] H. Martini, S. Wu, Radial projections of bisectors in Minkowski spaces, *Extracta Math.* **23** (1) (2008), 7 – 28.

[17] A. Skorik, M. Zaidenberg, On isometric reflections in Banach spaces, *Mat. Fiz. Anal. Geom.* **4** (1-2) (1997), 212 – 247.

[18] A.C. Thompson, "Minkowski Geometry", Encyclopedia of Mathematics and its Applications, 63, Cambridge University Press, Cambridge, 1996.