Uniform consistency of kernel–type estimators and conditional U–statistics with general bandwidths

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Abstract

Let $(X,Y),(X_1,Y_1),\ldots,(X_n,Y_n)$ be i.i.d. random variables and denote by f_X the (unknown) marginal density function of X. For practical use, it is often important to be able to estimate this density, as well as the regression function $m(t)=\mathbb{E}[Y|X=t]$. A popular class of estimators consists of kernel–type estimators, which have been intensively studied for a long time. Some well–known examples are the Nadaraya–Watson estimator and the local polynomial regression function estimators.

We start by giving some recent uniform consistency results for kernel–type estimators of f_X and $m(t)$, with a concise overview of the used methodology. The most recent results are shown to be valid uniformly in bandwidth as well, implying that the consistency is preserved when the bandwidth is chosen depending upon the datatset and/or the location. This kind of result is both from theoretical as practical point of view a considerable improvement.

Next, we look at conditional U–statistics, which were introduced by Stute to estimate the general regression function $m_\varphi(\mathbf{t})=\mathbb{E}[\varphi(Y_1,\ldots,Y_m)|(X_1,\ldots,X_m)=\mathbf{t}]$, and for which pointwise consistency was proved in 1991. We are now interested in the uniform convergence of these estimators, so a strong uniform consistency result is established, where a uniformity in the bandwidth is considered as well. In addition, the corresponding convergence rates can be derived from this result.

References

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