Test of fit for symmetric variance gamma distributions

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Keywords: Variance Gamma Distribution, Characteristic Function, EM - type algorithm, Goodness of Fit

Abstract

The aim of this paper is to provide goodness-of-fit tests for the symmetric normal variance gamma distribution (SNVG). The NVG distribution is defined as a mixture of a normal distribution with a gamma distribution. The SNVG distribution is a three - parameter model, denoted by $SNVG(\delta, c, \lambda)$ which depends on a location parameter $\delta \in \mathbf{R}$, a scale parameter $c > 0$, and a shape parameter $\lambda > 0$.

Suppose that on the basis of independent copies $X_1, X_2, \ldots, X_n$, of a random variable X we wish to test the null hypothesis

H_0: The law of X is $SNVG(\delta, c, \lambda)$ for some $\delta \in \mathbf{R}$, $\sigma > 0$ and $\lambda > 0$.

We study a new family of omnibus tests of H_0 based on the empirical characteristic function (CF). Despite the fact that the density function of X is complicated, the CF, $\phi(t) = \mathbf{E}(e^{itX})$ of X is simply

$$\phi(t; \delta, c, \lambda) = e^{i\delta t}(1 + c^2 t^2)^{-\lambda}. \quad (1)$$

Its main characteristics are that the empirical CF is an unbiased and consistent estimator of the CF population and that, under specific sampling situations - including the present one - is more convenient to employ methods based on it, rather than classical methods such as moment based or likelihood inference methods. The "Fourier approach" has been proved to be a simple and very powerful tool for statistical inference, particularly in goodness - of - fit problems. Many inference procedures based on the empirical characteristic function have appeared lately in the literature.

A critical issue is the estimation of the parameters. Due to the complexity of the density function a direct maximization of the likelihood function is difficult while the method of moments estimates do not converge rapidly. In order to exceed this problem we use an EM - type algorithm.

The results of a Monte Carlo study would be presented. The Monte Carlo study was implemented by drawing 1000 samples of size $n = 50$, $n = 100$ and $n = 200$. However, since the null distribution of the test statistic depends on the value of the parameter λ, which is unknown, we resort to a parametric bootstrap procedure.