On an Adaptive Goodness–of–Fit test with Finite Sample Validity for Random Design Regression Models

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Abstract

Given an i.i.d. sample $\{(X_i, Y_i)\}_{i\in\{1,\ldots,n\}}$ from the random design regression model $Y = f(X) + \epsilon$ with $(X, Y) \in [0, 1] \times [-M, M]$, we consider the problem of testing the (simple) null hypothesis "$f = f_0$", against the alternative "$f \neq f_0$" from a non-asymptotic point of view and for a fixed $f_0 \in \mathsf{L}^2([0, 1], G_X)$, where $G_X(\cdot)$ denotes the (known) marginal distribution of the design variable X. The procedure proposed is an adaptation to the regression setting of a multiple testing technique introduced by Fromont and Laurent (2006), and it amounts to consider a suitable collection of unbiased estimators of the L^2–distance $\mathsf{d}_2(f, f_0) = \int [f(x) - f_0(x)]^2 \mathrm{d}G_X(x)$, rejecting the null hypothesis when at least one of them is greater than its $(1 - u_\alpha)$ quantile, with u_α calibrated to obtain a level–α test. These estimators are built upon the *warped wavelet* system recently introduced by Picard and Kerkyacharian (2004), and the resulting testing procedure turns out to be adaptive over a particular collection of approximation spaces linked to the classical Besov spaces. Possible extensions of the proposed procedure to other settings (e.g. two–sample problems, unknown design density, composite-hypotheses), and an analogous goodness–of–fit test on the sphere based on a new type of spherical wavelets, called needlets (Narcowich et al., 2006) will also be sketched.

References

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