On the principle of smooth fit in optimal stopping problems

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Abstract

Given a function G: $\mathbb{R}^n \to \mathbb{R}$ and a Markov process $(X_t)_{t\geq 0}$ consider the optimal stopping problem $V(x) = \sup_\tau \mathsf{E}_x G(X_\tau)$. It is well known that the optimal stopping time is given by $\tau^* = \inf\{t : X_t \in D\}$ where the set D is given by $D := \{x : V(x) = G(x)\}$ and V is the smallest superharmonic function that dominates G. This characterization could be reformulated as the free-boundary problem, where both V and D have to be determined. To get the explicit solution of the arising differential equations one has to impose additional boundary conditions on V. Often, this conditions follow from the principle of smooth fit that states $V' = G'$ on ∂D (in one-dimensional case) under various assumptions on G and X. This results are well-known and widely used, but only weakest of them could be extended directly to multi-dimensional case due to topological complications. Therefore, "smooth fit" is considered in topology weaker than standard topology of $\mathbb{R}^n$. In the article sufficient conditions are presented for this principle of smooth fit to hold in multi-dimensional case.

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References

[1] Gikhman I. I. *and* Skorohod A. V. (1975). *The Theory of Stochastic Processes.* Nauka

[2] Grigelionis, B. I. *and* Shiryaev, A. N. (1966) *On Stefan's problem and optimal stopping rules for Markov processes.* Theory Probab. Appl. 11, 541-558

[3] Shiryaev, A. N. *and* Peskir, G. (2006). *Optimal Stopping and Free-Boundary Problems.* Birkhauser