Gaussian Process Simulation with application of the Theory of Square-Gaussian Processes

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Abstract

In many applied areas which use theory of stochastic processes the problem arises to estimate probability that a random vector process $\overrightarrow{X}^T(t) = (X_1(t), X_2(t), \ldots, X_d(t))$ leaves some region on some interval of time. For example, some systems break off on the interval $[0, T]$ if

$$\sup_{0 \le t \le T} \sum_{k=1}^{d} c_k^2(t) X_k^2(t) > \varepsilon$$

or, in more general case, if

$$\sup_{t \in T} \overrightarrow{X}^T(t) A(t) \overrightarrow{X}(t) > \varepsilon,$$

where ε is a sufficiently large number, (T, ρ) is a metric space, $\overrightarrow{X} = (\overrightarrow{X}(t), t \in T)$ is a process that generates the system, $A(t)$ is a matrix (in most cases positive semidefinite). The process $\overrightarrow{X}(t)$ may be considered as Gaussian due to the central limit theorem. Thus the problem arises to estimate the probability

$$P\left\{\sup_{t \in T} \overrightarrow{X}^T(t) A(t) \overrightarrow{X}(t) > \varepsilon\right\},$$

or the probability

$$P\left\{\sup_{t \in T} |\overrightarrow{X}^T(t) A(t) \overrightarrow{X}(t) - \mathbf{E}\overrightarrow{X}^T(t) A(t) \overrightarrow{X}(t)| > \varepsilon\right\},$$

where $\overrightarrow{X}(t)$ is a Gaussian vector process and $A(t)$ is a symmetric matrix. The process $\overrightarrow{X}(t)$ is considered as centered one.

The estimates of such large deviation probability for square-Gaussian stochastic processes are obtained. This result is used for model construction of Gaussian stochastic process as input process of same system, taking into account output process, with given reliability and accuracy in Banach space.