monomialideal.lib

I. Bermejo
University of La Laguna
e-mail: ibermejo@ull.es

E. Garcia-Llorente
University of La Laguna
e-mail: evgarcia@ull.es

P. Gimenez
University of Valladolid
e-mail: pgimenez@agt.uva.es

Abstract

Monomial ideals in a polynomial ring $k[x_1, \ldots, x_x]$ over a field k are the ideals of $k[x_1, \ldots, x_n]$ generated by monomials. The minimal monomial generating set is unique. Monomial ideals are the simplest ideals one can work with in Commutative Algebra and, moreover, they play a central role in this area. Indeed, using Gröbner basis theory, many problems on polynomial ideals, like the computation of the Krull dimension or the Hilbert series of a quotient ring, can be solved handling with their monomial version. Another important characteristic of monomial ideals is their combinatorial structure. Monomial ideals can be related to other combinatorial structures like graphs, hypergraphs, simplicial complexes, etc. This correspondence permits to solve problems on monomial ideals using combinatorial techniques or, on the contrary, to use Commutative Algebra to solve problems in Combinatorics.

One of our motivations for doing this library comes from the work by Bermejo and Gimenez [3] where several methods for computing the Castelnuovo-Mumford regularity, or simply regularity, of an arbitrary homogeneous ideal are given. They reduce this computation to the computation of the regularity of a monomial ideal of a special kind called a monomial ideal of nested type. One of the methods for computing the regularity of a monomial ideal of nested type requires the knowledge of its irredundant irreducible decomposition. This method, which is probably the most efficient, has not been implemented in the SINGULAR library `mregular.lib` [4] because none of the methods for constructing the irredundant irreducible decomposition of a monomial ideal had been previously implemented in SINGULAR. This is the aim of our library `monomialideal.lib` [2] that contains the implementation of eight different methods for solving this problem. The first one only uses the characterization of irreducible monomial ideals, it is very easy but very inefficient since it generates many superfluous components that have to be removed. The other methods that we have implemented are based on works by Milowski [10], Gao and Zhu [7], Roune [11], [12] and Miller [9]. From the irreducible decomposition of a monomial ideal, one can easily obtain a minimal primary decomposition. This has also been implemented.

The library `monomialideal.lib` also contains a list of functions for computing with monomial ideals. The first one is a function which determines whether an ideal is monomial when the generating set is not monomial. There are also procedures for determining whether a monomial ideal is primary, prime, irreducible, artinian or generic. Finally, the library also includes procedures for computing the radical of a monomial ideal, and intersections and quotients of monomial ideals. A similar library is being implemented in CoCoA; see [6].

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