Hierarchical statistical models and special factorizations

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Abstract

Hierarchical statistical models, which include decomposable graphical models as a special case, are a very large class and there are many interesting sub-classes. There are two fundamental types. The first are standard polynomial regression models, where the terms in the model can be considered as univariate polynomial models in each factor with generalised interaction terms across factors. Models are give by a "staircase" of monomial terms, with an underlying simplicial structure, which is most clear in the square-free case. The second category is for log-linear categorical models where the same regression is used, but now for the log (joint) probability. Here, the square-free case corresponds to binary random variables and gives a special factorization of the joint probability and a toric ideal. In certain cases, such as decomposable graphical models, the factorisation is simple, but in general it is complex in structure and requires interpretation. This interpretation is even more difficult than the interpretation of interactions in the regression case. We study one particularly class, namely Lattice Conditional Independence models, where some additional progress can be made and there is an interpretation in terms of projection operators. Tools from algebra, include lattices, simplicial complexes and Betti numbers. The talk is based on work in progress and will point to a number of forthcoming manuscripts joint with other authors, in particular: Hugo Maruri-Aguilar, Eduardo Sáenze de Cabezón, Giovanni Pistone and Daniel Bruynooghe.