Resolución de ecuaciones

- Newton-Raphson: $x_{n+1} = x_n - f(x_n)/f'(x_n)$,

$$|\bar{x} - x_n| \leq \frac{M}{2m}|x_n - x_{n-1}|^2, \quad M = \max_{x \in [a,b]} |f''(x)|, \quad m = \min_{x \in [a,b]} |f'(x)|.$$

- Fórmula de extrapolación de Aitken: $c - x_n \approx \frac{\lambda_n}{1-\lambda_n}(x_n - x_{n-1})$,
$\lambda_n := \frac{x_n - x_{n-1}}{x_{n-1} - x_{n-2}}$.

Interpolación y aproximación

- Error de interpolación: $f(x) - P_n(x) = \frac{f^{(n+1)}}{(n+1)!}(x - x_0)(x - x_1)\ldots(x - x_n)$

- Lagrange: $L_i(x) = \prod_{j \neq i} \frac{(x - x_j)}{(x_i - x_j)}$.

- Diferencias divididas:

$$f[x_k] = f(x_k), f[x_i, x_{i+1}, \ldots, x_j] = \frac{f[x_{i+1}, x_{i+2}, \ldots, x_j] - f[x_i, x_{i+1}, \ldots, x_{j-1}]}{x_j - x_i},$$

$$f[x_k, \overset{m}{\ldots}, x_k] = \frac{f^{m-1)}(x_k)}{(m-1)!}.$$

Integración

- Derivadas:

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}, \quad f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}.$$

- Trapecio: $\int_a^b f(x)\,dx = \frac{b-a}{2}(f(a) + f(b))$. Error: $E = -\frac{(b-a)^3}{12}f''(\xi)$.

- Simpson: $\int_a^b f(x)\,dx = \frac{b-a}{6}(f(a) + 4f(x_1) + f(b))$. Error: $E = -\frac{(b-a)^5}{720}f^{(4)}(\xi)$.

- Error métodos compuestos:
Trapecio: $-\frac{f''(\xi)}{12}(b-a)h^2$, Simpson: $-\frac{f^{(4)}(\xi)}{180}h^4(b-a)$.

- Cuadratura adaptativa: $E_1 = (S(f, a, c) + S(f, c, b) - S(f, a, b))/15$.

- Euler: $x_{k+1} = x_k + hf(t_k, x_k)$.

- Euler modificado: $x_{k+1} = x_k + \frac{h}{2}(f(t_k, x_k) + f(t_k + h, x_k + hf(t_k, x_k)))$

- Runge-Kutta: $x_{k+1} = x_k + \frac{h}{6}(f_{k_1} + 2f_{k_2} + 2f_{k_3} + f_{k_4})$,

$$f_{k_1} = f(t_k, x_k), \quad f_{k_2} = f(t_k + h/2, x_k + hf_{k_1}/2),$$
$$f_{k_3} = f(t_k + h/2, x_k + hf_{k_2}/2), \quad f_{k_4} = f(t_k + h, x_k + hf_{k_3})$$