Cálculo Numérico y Estadística

Primera parte: Cálculo Numérico

José Luis Bravo

Curso 2019/2020
Segundo semestre

Método de eliminación de Gauss:

El método de eliminación de Gauss es un método directo de resolución de un sistema lineal de n ecuaciones con n incógnitas (que suponemos no singular)

$$\begin{array}{c} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{array}$$

que consiste en transformar el sistema original en uno equivalente triangular superior,

$$\begin{array}{rcrcl} a'_{11}x_1 + a'_{12}x_2 + & \cdots & + a'_{1n}x_n & = & b'_1 \\ a'_{22}x_2 + & \cdots & + a'_{2n}x_n & = & b'_2 \\ & \cdots\cdots\cdots & & & \\ & & a'_{n-1\,n-1}x_{n-1} + a'_{n-1\,n}x_n & = & b'_{n-1} \\ & & a'_{nn}x_n & = & b'_n \end{array}$$

cuya resolución (comenzando por la última ecuación) resulta mucho más sencilla.

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$$\begin{array}{l} 5x_1 + 7x_2 + 6x_3 = 14 \\ 3x_1 - 4x_2 + 2x_3 = 9 \\ 2x_1 + 5x_2 + 7x_3 = 10 \end{array}$$

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$$x_3 = \frac{187}{171}, \; x_2 = -\frac{49}{171}, \; x_1 = \frac{17}{9}$$

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ec 3: $2x_1 + 5x_2 + 7x_3 = 10 \quad \rightarrow \quad (\text{ec } 3) - \frac{a_{31}}{a_{11}}(\text{ec } 1) \rightarrow \quad \frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}$

$$\{3x_1 - 4x_2 + 2x_3 = 9\} - \frac{3}{5}\{5x_1 + 7x_2 + 6x_3 = 14\} \rightarrow \frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}$$

$$\{2x_1 + 5x_2 + 7x_3 = 10\} - \frac{2}{5}\{5x_1 + 7x_2 + 6x_3 = 14\} \rightarrow \frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}$$

$\frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5} \quad \rightarrow \quad \frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}$

$\frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5} \quad \rightarrow \quad \frac{171}{41}x_3 = \frac{187}{41}$

$$\left\{\frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}\right\} - \frac{\frac{11}{5}}{\frac{-41}{5}}\left\{\frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}\right\} \rightarrow \frac{171}{41}x_3 = \frac{187}{41}$$

Resolvemos el sistema triangular:

Veamos todo el proceso:

ec 1: $5x_1 + 7x_2 + 6x_3 = 14 \quad \rightarrow \quad 5x_1 + 7x_2 + 6x_3 = 14$

ec 2: $3x_1 - 4x_2 + 2x_3 = 9 \quad \rightarrow \quad (\text{ec } 2) - \frac{a_{21}}{a_{11}}(\text{ec } 1) \rightarrow \quad \frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}$

ec 3: $2x_1 + 5x_2 + 7x_3 = 10 \quad \rightarrow \quad (\text{ec } 3) - \frac{a_{31}}{a_{11}}(\text{ec } 1) \rightarrow \quad \frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}$

$$\{3x_1 - 4x_2 + 2x_3 = 9\} - \frac{3}{5}\{5x_1 + 7x_2 + 6x_3 = 14\} \rightarrow \frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}$$

$$\{2x_1 + 5x_2 + 7x_3 = 10\} - \frac{2}{5}\{5x_1 + 7x_2 + 6x_3 = 14\} \rightarrow \frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}$$

$$\frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5} \quad \rightarrow \quad \frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}$$

$$\frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5} \quad \rightarrow \quad \frac{171}{41}x_3 = \frac{187}{41}$$

$$\left\{\frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}\right\} - \frac{\frac{11}{5}}{\frac{-41}{5}}\left\{\frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}\right\} \rightarrow \frac{171}{41}x_3 = \frac{187}{41}$$

Resolvemos el sistema triangular:

ec 3: $x_3 = \frac{187}{171}$

Veamos todo el proceso:

ec 1: $5x_1 + 7x_2 + 6x_3 = 14 \quad \rightarrow \quad 5x_1 + 7x_2 + 6x_3 = 14$

ec 2: $3x_1 - 4x_2 + 2x_3 = 9 \quad \rightarrow \quad (\text{ec } 2) - \frac{a_{21}}{a_{11}}(\text{ec } 1) \rightarrow \quad \frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}$

ec 3: $2x_1 + 5x_2 + 7x_3 = 10 \quad \rightarrow \quad (\text{ec } 3) - \frac{a_{31}}{a_{11}}(\text{ec } 1) \rightarrow \quad \frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}$

$$\{3x_1 - 4x_2 + 2x_3 = 9\} - \frac{3}{5}\{5x_1 + 7x_2 + 6x_3 = 14\} \rightarrow \frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}$$

$$\{2x_1 + 5x_2 + 7x_3 = 10\} - \frac{2}{5}\{5x_1 + 7x_2 + 6x_3 = 14\} \rightarrow \frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}$$

$$\frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5} \quad \rightarrow \quad \frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}$$

$$\frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5} \quad \rightarrow \quad \frac{171}{41}x_3 = \frac{187}{41}$$

$$\left\{\frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}\right\} - \frac{\frac{11}{5}}{\frac{-41}{5}}\left\{\frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}\right\} \rightarrow \frac{171}{41}x_3 = \frac{187}{41}$$

Resolvemos el sistema triangular:

ec 3: $x_3 = \frac{187}{171}$

ec 2: $x_2 = \frac{-5}{41}\left(\frac{8}{5}x_3 + \frac{3}{5}\right) = \frac{-1}{41}\left(8\frac{187}{171} + 3\right) = \frac{-49}{171}$

Veamos todo el proceso:

ec 1: $5x_1 + 7x_2 + 6x_3 = 14 \quad \rightarrow \quad 5x_1 + 7x_2 + 6x_3 = 14$

ec 2: $3x_1 - 4x_2 + 2x_3 = 9 \quad \rightarrow \quad (\text{ec } 2) - \frac{a_{21}}{a_{11}}(\text{ec } 1) \rightarrow \quad \frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}$

ec 3: $2x_1 + 5x_2 + 7x_3 = 10 \quad \rightarrow \quad (\text{ec } 3) - \frac{a_{31}}{a_{11}}(\text{ec } 1) \rightarrow \quad \frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}$

$$\left\{3x_1 - 4x_2 + 2x_3 = 9\right\} - \frac{3}{5}\left\{5x_1 + 7x_2 + 6x_3 = 14\right\} \rightarrow \frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}$$

$$\left\{2x_1 + 5x_2 + 7x_3 = 10\right\} - \frac{2}{5}\left\{5x_1 + 7x_2 + 6x_3 = 14\right\} \rightarrow \frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}$$

$$\frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5} \quad \rightarrow \quad \frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}$$

$$\frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5} \quad \rightarrow \quad \frac{171}{41}x_3 = \frac{187}{41}$$

$$\left\{\frac{11}{5}x_2 + \frac{23}{5}x_3 = \frac{22}{5}\right\} - \frac{\frac{11}{5}}{\frac{-41}{5}}\left\{\frac{-41}{5}x_2 - \frac{8}{5}x_3 = \frac{3}{5}\right\} \rightarrow \frac{171}{41}x_3 = \frac{187}{41}$$

Resolvemos el sistema triangular:

ec 3: $x_3 = \frac{187}{171}$

ec 2: $x_2 = \frac{-5}{41}\left(\frac{8}{5}x_3 + \frac{3}{5}\right) = \frac{-1}{41}\left(8\frac{187}{171} + 3\right) = \frac{-49}{171}$

ec 1: $x_1 = \frac{1}{5}(-7x_2 - 6x_3 + 14) = \frac{1}{5}\left(-7\frac{-49}{171} - 6\frac{187}{171} + 14\right) = \frac{17}{9}$

Otro ejemplo:

Otro ejemplo:

$$\begin{aligned} 3x_2 - 6x_3 + x_4 &= 8 \\ 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned}$$

Otro ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \quad \rightarrow \quad \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array}$$

Otro ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \quad \rightarrow \quad \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array}$$

$$3x_2 - 6x_3 + x_4 = 8 \quad \rightarrow \left\{3x_2 - 6x_3 + x_4 = 8\right\} - \frac{0}{2}\left\{2x_1 + x_2 - x_3 + 2x_4 = -2\right\}$$

$$\rightarrow 3x_2 - 6x_3 + x_4 = 8$$

Otro ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \quad \rightarrow \quad \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array}$$

$$3x_2 - 6x_3 + x_4 = 8 \quad \rightarrow \left\{3x_2 - 6x_3 + x_4 = 8\right\} - \frac{0}{2}\left\{2x_1 + x_2 - x_3 + 2x_4 = -2\right\}$$

$$\rightarrow 3x_2 - 6x_3 + x_4 = 8$$

$$-x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \quad \rightarrow \left\{-x_1 + 3x_2 - 5x_3 - 6x_4 = 3\right\}$$

$$- \frac{(-1)}{2}\left\{2x_1 + x_2 - x_3 + 2x_4 = -2\right\}$$

$$\rightarrow \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2$$

Otro ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array}$$

$$3x_2 - 6x_3 + x_4 = 8 \quad \rightarrow \left\{3x_2 - 6x_3 + x_4 = 8\right\} - \frac{0}{2}\left\{2x_1 + x_2 - x_3 + 2x_4 = -2\right\}$$

$$\rightarrow 3x_2 - 6x_3 + x_4 = 8$$

$$-x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \quad \rightarrow \left\{-x_1 + 3x_2 - 5x_3 - 6x_4 = 3\right\}$$

$$-\frac{(-1)}{2}\left\{2x_1 + x_2 - x_3 + 2x_4 = -2\right\}$$

$$\rightarrow \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2$$

$$x_1 + 4x_2 - 3x_3 + x_4 = 4 \quad \rightarrow \left\{x_1 + 4x_2 - 3x_3 + x_4 = 4\right\}$$

$$-\frac{1}{2}\left\{2x_1 + x_2 - x_3 + 2x_4 = -2\right\}$$

$$\rightarrow \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5$$

Otro ejemplo:

$$\begin{aligned} 3x_2 - 6x_3 + x_4 &= 8 \\ 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \quad \rightarrow \quad \begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \quad \rightarrow$$

$$\begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 &= 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 &= 5 \end{aligned}$$

Otro ejemplo:

$$\begin{aligned} 3x_2 - 6x_3 + x_4 &= 8 \\ 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \quad \rightarrow \quad \begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \quad \rightarrow$$

$$\begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ \tfrac{7}{2}x_2 - \tfrac{11}{2}x_3 - 5x_4 &= 2 \\ \tfrac{7}{2}x_2 - \tfrac{5}{2}x_3 &= 5 \end{aligned}$$

$$\tfrac{7}{2}x_2 - \tfrac{11}{2}x_3 - 5x_4 = 2 \quad \rightarrow \left\{\tfrac{7}{2}x_2 - \tfrac{11}{2}x_3 - 5x_4 = 2\right\} - \tfrac{\frac{7}{2}}{3}\left\{3x_2 - 6x_3 + x_4 = 8\right\}$$

$$\rightarrow \tfrac{3}{2}x_3 - \tfrac{37}{6}x_4 = -\tfrac{22}{3}$$

Otro ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \quad \rightarrow \quad \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \quad \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array}$$

$$\frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \quad \rightarrow \left\{\frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2\right\} - \frac{\frac{7}{2}}{3}\left\{3x_2 - 6x_3 + x_4 = 8\right\}$$

$$\rightarrow \frac{3}{2}x_3 - \frac{37}{6}x_4 = -\frac{22}{3}$$

$$\frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \quad \rightarrow \left\{\frac{7}{2}x_2 - \frac{5}{2}x_3 = 5\right\} - \frac{\frac{7}{2}}{3}\left\{3x_2 - 6x_3 + x_4 = 8\right\}$$

$$\rightarrow \frac{9}{2}x_3 - \frac{7}{6}x_4 = -\frac{13}{3}$$

Otro ejemplo:

$$\begin{aligned} 3x_2 - 6x_3 + x_4 &= 8 \\ 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \quad \rightarrow \quad \begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \quad \rightarrow$$

$$\begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ \tfrac{7}{2}x_2 - \tfrac{11}{2}x_3 - 5x_4 &= 2 \\ \tfrac{7}{2}x_2 - \tfrac{5}{2}x_3 &= 5 \end{aligned} \quad \rightarrow \quad \begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ \tfrac{3}{2}x_3 - \tfrac{37}{6}x_4 &= -\tfrac{22}{3} \\ \tfrac{9}{2}x_3 - \tfrac{7}{6}x_4 &= -\tfrac{13}{3} \end{aligned}$$

$$\tfrac{7}{2}x_2 - \tfrac{11}{2}x_3 - 5x_4 = 2 \quad \rightarrow \left\{\tfrac{7}{2}x_2 - \tfrac{11}{2}x_3 - 5x_4 = 2\right\} - \frac{\frac{7}{2}}{3}\left\{3x_2 - 6x_3 + x_4 = 8\right\}$$

$$\rightarrow \tfrac{3}{2}x_3 - \tfrac{37}{6}x_4 = -\tfrac{22}{3}$$

$$\tfrac{7}{2}x_2 - \tfrac{5}{2}x_3 = 5 \quad \rightarrow \left\{\tfrac{7}{2}x_2 - \tfrac{5}{2}x_3 = 5\right\} - \frac{\frac{7}{2}}{3}\left\{3x_2 - 6x_3 + x_4 = 8\right\}$$

$$\rightarrow \tfrac{9}{2}x_3 - \tfrac{7}{6}x_4 = -\tfrac{13}{3}$$

Otro ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \quad \rightarrow \quad \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \quad \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \quad \rightarrow \quad \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{3}{2}x_3 - \frac{37}{6}x_4 = -\frac{22}{3} \\ \frac{9}{2}x_3 - \frac{7}{6}x_4 = -\frac{13}{3} \end{array}$$

$$\frac{9}{2}x_3 - \frac{7}{6}x_4 = -\frac{13}{3} \quad \rightarrow \left\{\frac{9}{2}x_3 - \frac{7}{6}x_4 = -\frac{13}{3}\right\} - \frac{\frac{9}{2}}{\frac{3}{2}}\left\{\frac{3}{2}x_3 - \frac{37}{6}x_4 = -\frac{22}{3}\right\}$$

$$\rightarrow \frac{52}{3}x_4 = \frac{53}{3}$$

Otro ejemplo:

$$\begin{aligned} 3x_2 - 6x_3 + x_4 &= 8 \\ 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \quad \to \quad \begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \quad \to$$

$$\begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ \tfrac{7}{2}x_2 - \tfrac{11}{2}x_3 - 5x_4 &= 2 \\ \tfrac{7}{2}x_2 - \tfrac{5}{2}x_3 &= 5 \end{aligned} \quad \to \quad \begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ \tfrac{3}{2}x_3 - \tfrac{37}{6}x_4 &= -\tfrac{22}{3} \\ \tfrac{9}{2}x_3 - \tfrac{7}{6}x_4 &= -\tfrac{13}{3} \end{aligned} \quad \to$$

$$\begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ \tfrac{3}{2}x_3 - \tfrac{37}{6}x_4 &= -\tfrac{22}{3} \\ \tfrac{52}{3}x_4 &= \tfrac{53}{3} \end{aligned}$$

$$\frac{9}{2}x_3 - \frac{7}{6}x_4 = -\frac{13}{3} \quad \to \left\{\frac{9}{2}x_3 - \frac{7}{6}x_4 = -\frac{13}{3}\right\} - \frac{\frac{9}{2}}{\frac{3}{2}}\left\{\frac{3}{2}x_3 - \frac{37}{6}x_4 = -\frac{22}{3}\right\}$$

$$\to \frac{52}{3}x_4 = \frac{53}{3}$$

Otro ejemplo:

$$\begin{aligned} 3x_2 - 6x_3 + x_4 &= 8 \\ 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \quad \to \quad \begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \quad \to$$

$$\begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ \tfrac{7}{2}x_2 - \tfrac{11}{2}x_3 - 5x_4 &= 2 \\ \tfrac{7}{2}x_2 - \tfrac{5}{2}x_3 &= 5 \end{aligned} \quad \to \quad \begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ \tfrac{3}{2}x_3 - \tfrac{37}{6}x_4 &= -\tfrac{22}{3} \\ \tfrac{9}{2}x_3 - \tfrac{7}{6}x_4 &= -\tfrac{13}{3} \end{aligned} \quad \to$$

$$\begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ \tfrac{3}{2}x_3 - \tfrac{37}{6}x_4 &= -\tfrac{22}{3} \\ \tfrac{52}{3}x_4 &= \tfrac{53}{3} \end{aligned} \quad \to$$

$$x_4 = \frac{53}{52}, \quad x_3 = -\frac{109}{156}, \quad x_2 = \frac{145}{156}, \quad x_1 = -\frac{17}{6}$$

Ejercicio: Comprobar que para resolver el siguiente sistema con el método de Gauss se van dando los pasos intermedios que aparecen a continuación:

$$\begin{aligned} 3x_1 - 2x_2 + x_3 - 4x_4 &= 1 \\ x_1 + x_2 + x_3 + x_4 &= 7 \\ -x_1 + 3x_2 - x_3 + x_4 &= 12 \\ 2x_1 + 2x_2 + 3x_3 - x_4 &= 5 \end{aligned} \quad \rightarrow \quad \begin{aligned} 3x_1 - 2x_2 + x_3 - 4x_4 &= 1 \\ \tfrac{5}{3}x_2 + \tfrac{2}{3}x_3 + \tfrac{7}{3}x_4 &= \tfrac{20}{3} \\ \tfrac{7}{3}x_2 - \tfrac{2}{3}x_3 - \tfrac{1}{3}x_4 &= \tfrac{37}{3} \\ \tfrac{10}{3}x_2 + \tfrac{7}{3}x_3 + \tfrac{5}{3}x_4 &= \tfrac{13}{3} \end{aligned} \quad \rightarrow$$

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$$x_4 = \frac{19}{14}, \; x_3 = -\frac{69}{14}, \; x_2 = \frac{57}{14}, \; x_1 = \frac{13}{2}$$

Ejemplo "con errores":

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ec 1: $3x_1 + 3x_2 = 3$

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Resolvemos el sistema triangular:

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