Cálculo Numérico y Estadística

Primera parte: Cálculo Numérico

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Curso 2019/2020
Segundo semestre

Interpolación de Hermite

Si cuando queremos que el polinomio de interpolación no solamente pase por determinados puntos, sino que en esos puntos algunas de sus derivadas tomen ciertos valores, estamos hablando de interpolación de Hermite. Por ejemplo, supongamos que queremos calcular un polinomio $P(x)$, del menor grado posible, que coincida con los datos de la función $f(x)$ de la siguiente tabla

i	x_i	$f(x_i)$	$f'(x_i)$	$f''(x_i)$
0	0	2	3	
1	0.5	-1		
2	1	1	2	-1
3	1.5	0.5		

Es decir, queremos que $P(x)$ verifique las siguientes condiciones:

$$P(0) = 2, \quad P'(0) = 3,$$
$$P(0.5) = -1,$$
$$P(1) = 1, \quad P'(1) = 2, \quad P''(1) = -1,$$
$$P(1.5) = 0.5.$$

Veremos que este polinomio podemos calcularlo en la forma de Newton haciendo pequeñas modificaciones.

Antes de nada, debemos tener en cuenta la siguiente proposición:

Proposición: Sean $x_0, x_1, \ldots, x_n$, $n+1$ puntos distintos. Dados los valores

$$\begin{array}{ccccc}
x_0 & x_1 & x_2 & \ldots & x_n \\
y_0^{(0)} & y_1^{(0)} & y_2^{(0)} & \ldots & y_n^{(0)} \\
y_0^{(1)} & y_1^{(1)} & y_2^{(1)} & \ldots & y_n^{(1)} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
y_0^{(n_0-1)} & y_1^{(n_1-1)} & y_2^{(n_2-1)} & \ldots & y_n^{(n_n-1)}
\end{array}$$

existe un único polinomio $P(x)$ de grado $\leq n_0 + n_1 + n_2 + \cdots + n_n - 1$ (n$^\circ$ de condiciones -1) tal que

$$\begin{array}{lllll}
P(x_0) = y_0^{(0)}, & P'(x_0) = y_0^{(1)}, & P''(x_0) = y_0^{(2)}, & \ldots, & P^{n_0-1)}(x_0) = y_0^{(n_0-1)} \\
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> Por tanto, ya sabemos que va a existir un polinomio $P(x)$ de grado $\leq$? que verifica
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> $$\begin{array}{lll} P(0) = 2, & P'(0) = 3, & \\ P(0.5) = -1, & & \\ P(1) = 1, & P'(1) = 2, & P''(1) = -1, \\ P(1.5) = 0.5. & & \end{array}$$

existe un único polinomio $P(x)$ de grado $\leq n_0 + n_1 + n_2 + \cdots + n_n - 1$ (n° de condiciones -1) tal que

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Por tanto, ya sabemos que va a existir un polinomio $P(x)$ de grado $\leq$ 6 que verifica

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Debemos, también, extender la definición de diferencias divididas. Recuérdese que las definimos como

$$f[x_0, x_1] = \frac{f(x_0) - f(x_1)}{x_0 - x_1}$$

$$f[x_0, x_1, \ldots, x_m] = \frac{f[x_0, x_1, \ldots, x_{m-1}] - f[x_1, x_2, \ldots, x_m]}{x_0 - x_m}$$

siendo $x_0, x_1, \ldots, x_n$ puntos distintos entre si. Con esta definición no tiene sentido, por ejemplo, calcular $f[2, 2]$.

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Por ejemplo,

$$f[2,2] = f'(2), \qquad f[2,2,2] = \frac{f''(2)}{2}, \qquad f[2,2,2,2] = \frac{f^{3)}(2)}{6}, \qquad \ldots$$

Ejemplo: Para calcular el polinomio $P(x)$ de grado ≤ 6 que verifica:

$P(0) = 2,\qquad P'(0) = 3,$

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hacemos lo siguiente: cada x_i lo repetimos tantas veces como condiciones haya sobre él y construimos la tabla de las diferencias divididas en la forma habitual:

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
1	0	2						
2	0.5	-1						
3	1	1						
4	1	1						
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$P[x_0, x_1] =$

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$$\frac{P(0) - P(0.5)}{0 - 0.5} = -6$$

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0	0	2						
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$P(1.5) = 0.5.$

$P[x_4, x_5] = P[1, 1] = P'(1) = 2$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
1	0	2						
			-6					
2	0.5	-1						
			4					
3	1	1						
			2					
4	1	1						
			2					
5	1	1						
6	1.5	0.5						

$$P[x, x] = P'(x), \qquad P[x, x, x] = \frac{P''(x)}{2}, \qquad P[x, x, x, x] = \frac{P^{3)}(x)}{6}, \qquad \ldots$$

Ejemplo: Para calcular el polinomio $P(x)$ de grado ≤ 6 que verifica:

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$P(1.5) = 0.5.$

$P[x_5, x_6] =$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2	3					
1	0	2	-6					
2	0.5	-1	4					
3	1	1	2					
4	1	1	2					
5	1	1	?					
6	1.5	0.5						

$$P[x,x] = P'(x), \qquad P[x,x,x] = \frac{P''(x)}{2}, \qquad P[x,x,x,x] = \frac{P^{3)}(x)}{6}, \qquad \ldots$$

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$$P[x_5, x_6] = P[1, 1.5] = \frac{P(1) - P(1.5)}{1 - 1.5} = -1$$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
1	0	2						
			-6					
2	0.5	-1						
			4					
3	1	1						
			2					
4	1	1						
			2					
5	1	1						
			?					
6	1.5	0.5						

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i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
1	0	2						
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			2					
4	1	1						
			2					
5	1	1						
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$P[x_0, x_1, x_2] =$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
1	0	2		?				
			-6					
2	0.5	-1						
			4					
3	1	1						
			2					
4	1	1						
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$$P[x_0, x_1, x_2] = P[0,\ 0,\ 0.5] = \frac{P[0,\ 0] - P[0,\ 0.5]}{0 - 0.5} = \frac{3-(-6)}{0-0.5} = -18$$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
1	0	2		?				
			−6					
2	0.5	−1						
			4					
3	1	1						
			2					
4	1	1						
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5	1	1						
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1	0	2	-6	-18				
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5	1	1	-1					
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$P(1.5) = 0.5.$

$P[x_1, x_2, x_3] =$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2	3					
1	0	2	-6	-18				
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$P[x_1, x_2, x_3] = P[0,\ 0.5,\ 1] =$

$$\frac{P[0,\ 0.5] - P[0.5,\ 1]}{0 - 1} = \frac{-6 - 4}{0 - 1} = 10$$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
1	0	2		-18				
			-6					
2	0.5	-1		?				
			4					
3	1	1						
			2					
4	1	1						
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5	1	1						
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0	0	2						
			3					
1	0	2		-18				
			-6					
2	0.5	-1		10				
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3	1	1						
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i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
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1	0	2		-18				
			-6					
2	0.5	-1		10				
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			2					
4	1	1						
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$P[x_2, x_3, x_4] = P[0.5,\ 1,\ 1] =$

$$\frac{P[0.5,\ 1] - P[1,\ 1]}{0.5 - 1} = \frac{4 - 2}{0.5 - 1} = -4$$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
1	0	2		−18				
			−6					
2	0.5	−1		10				
			4					
3	1	1		?				
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0	0	2						
			3					
1	0	2		−18				
			−6					
2	0.5	−1		10				
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i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
1	0	2		-18				
			-6					
2	0.5	-1		10				
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			3					
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			-6					
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			-6					
2	0.5	-1		10				
			4					
3	1	1		-4				
			2					
4	1	1		$-\frac{1}{2}$				
			2					
5	1	1						
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i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
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3	1	1		-4				
			2					
4	1	1		$-\frac{1}{2}$				
			2					
5	1	1		?				
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			3					
1	0	2		-18				
			-6					
2	0.5	-1		10				
			4					
3	1	1		-4				
			2					
4	1	1		$-\frac{1}{2}$				
			2					
5	1	1		?				
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			3					
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2	0.5	-1		10				
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$P[x_0, x_1, x_2, x_3] =$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
1	0	2		-18				
			-6		?			
2	0.5	-1		10				
			4					
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$P(1.5) = 0.5$.

$$P[x_0, x_1, x_2, x_3] = P[0,\ 0,\ 0.5,\ 1] = \frac{P[0,\ 0,\ 0.5] - P[0,\ 0.5,\ 1]}{0-1} = \frac{-18-10}{0-1} = 28$$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
1	0	2		-18				
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$P(1.5) = 0.5.$

$$P[x_0, x_1, x_2, x_3] = P[0,\ 0,\ 0.5,\ 1] = \frac{P[0,\ 0,\ 0.5] - P[0,\ 0.5,\ 1]}{0 - 1} = \frac{-18 - 10}{0 - 1} = 28$$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
1	0	2		-18				
			-6		28			
2	0.5	-1		10				
			4					
3	1	1		-4				
			2					
4	1	1		$-\frac{1}{2}$				
			2					
5	1	1		-6				
			-1					
6	1.5	0.5						

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$P[x_1, x_2, x_3, x_4] =$

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0	0	2						
			3					
1	0	2		-18				
			-6		28			
2	0.5	-1		10				
			4		?			
3	1	1		-4				
			2					
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1	0	2		-18				
			-6		28			
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3	1	1		-4				
			2					
4	1	1		$-\frac{1}{2}$				
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i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
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i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
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i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
0	0	2						
			3					
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$P[x_3, x_4, x_5, x_6] =$

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$P[0,\ 0,\ 0.5,\ 1,\ 1] =$

$\frac{P[0,0,0.5,1]-P[0,0.5,1,1]}{0-1}$

$= \frac{28-(-14)}{0-1} = -42$

i	x_i	$P(x_i)$	$P[x_i, x_j]$	$P[(3)]$	$P[(4)]$	$P[(5)]$	$P[(6)]$	$P[(7)]$
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$P[0,\ 0,\ 0.5,\ 1,\ 1] =$
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$P[0,\ 0.5,\ 1,\ 1,\ 1] =$

$\frac{P[0,0.5,1,1]-P[0.5,1,1,1]}{0-1}$

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$\frac{P[0.5,1,1,1]-P[1,1,1,1.5]}{0.5-1.5}$

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$P[0.5,\ 1,\ 1,\ 1,\ 1.5] =$

$\frac{P[0.5,1,1,1]-P[1,1,1,1.5]}{0.5-1.5}$

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$\frac{P[0,0.5,1,1,1]-P[0.5,1,1,1,1.5]}{0-1.5}$
$= \frac{21-(-18)}{0-1.5} = -26$

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$\frac{P[0, 0.5, 1, 1, 1] - P[0.5, 1, 1, 1, 1.5]}{0 - 1.5}$

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Finalmente, $P(x)$ lo calculamos en la forma de Newton como

$$P(x) = P(x_0) + P[x_0, x_1](x - x_0) + P[x_0, x_1, x_2](x - x_0)(x - x_1) + \cdots$$
$$+ P[x_0, x_1, \ldots, x_6](x - x_0)(x - x_1)\cdots(x - x_5) =$$

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$$2 + 3(x-0) - 18(x-0)(x-0) + 28(x-0)(x-0)(x-0.5) - 42(x-0)(x-0)(x-0.5)(x-1)$$
$$+ 63(x-0)(x-0)(x-0.5)(x-1)(x-1) - \tfrac{178}{3}(x-0)(x-0)(x-0.5)(x-1)(x-1)(x-1)$$

$$P(x) = -\frac{178}{3}x^6 + \frac{812}{3}x^5 - \frac{933}{2}x^4 + \frac{1096}{3}x^3 - \frac{685}{6}x^2 + 3x + 2$$

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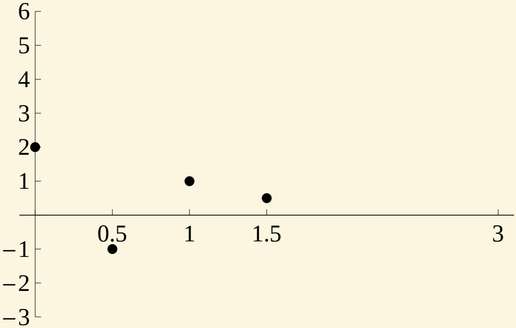

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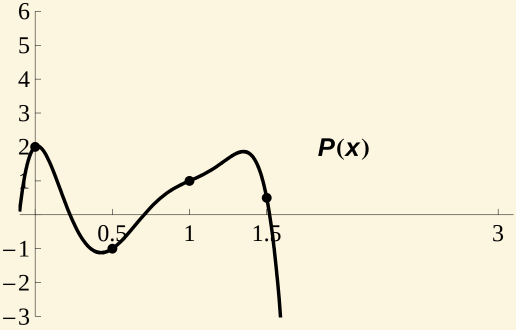

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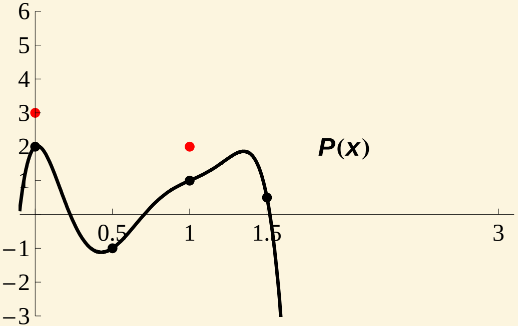

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$$P'(x) = -356x^5 + \frac{4060}{3}x^4 - 1866x^3 + 1096x^2 - \frac{685}{3}x + 3$$

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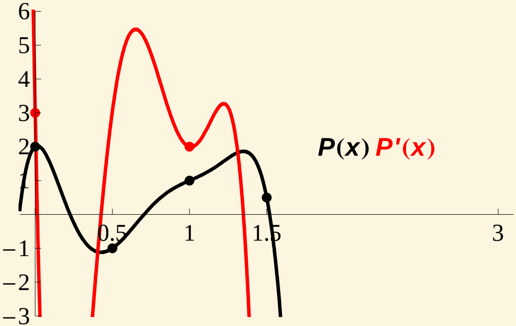

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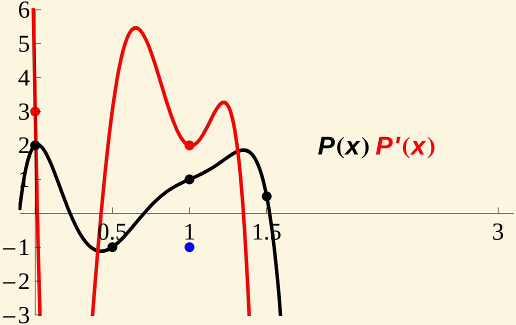

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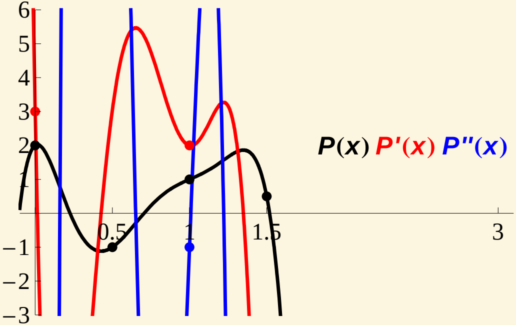

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