Cálculo Numérico y Estadística

Primera parte: Cálculo Numérico

José Luis Bravo

Curso 2019/2020
Segundo semestre

Método de eliminación de Gauss con elección de pivote (I):

Para minimizar los errores de redondeo que se cometen al realizar las operaciones necesarias para aplicar el método de Gauss debemos elegir el pivote de forma conveniente. En la aplicación del método de Gauss con "**pivote parcial**" procederemos de la siguiente forma:

En cada etapa del proceso, para pasar de un sistema de la forma

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned}$$

a otro equivalente de la forma

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ \bar{a}_{22}x_2 + \cdots + \bar{a}_{2n}x_n &= \bar{b}_2 \\ \cdots\cdots\cdots\cdots\cdots & \\ \bar{a}_{n2}x_2 + \cdots + \bar{a}_{nn}x_n &= \bar{b}_n \end{aligned}$$

reordenamos las ecuaciones originales para que el "**pivote**" a_{11} verifique

$$|a_{11}| = \max_{1 \le i \le n} |a_{i1}|.$$

Ejemplo:

Ejemplo:

$$\begin{aligned} 3x_2 - 6x_3 + x_4 &= 8 \\ 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned}$$

Ejemplo:

$$
\begin{aligned}
3x_2 - 6x_3 + x_4 &= 8 \\
2x_1 + x_2 - x_3 + 2x_4 &= -2 \\
-x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\
x_1 + 4x_2 - 3x_3 + x_4 &= 4
\end{aligned}
$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array}$$

$$3x_2 - 6x_3 + x_4 = 8 \quad \rightarrow \{3x_2 - 6x_3 + x_4 = 8\} - \tfrac{0}{2}\{2x_1 + x_2 - x_3 + 2x_4 = -2\}$$

$$\rightarrow 3x_2 - 6x_3 + x_4 = 8$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array}$$

$$3x_2 - 6x_3 + x_4 = 8 \quad \rightarrow \{3x_2 - 6x_3 + x_4 = 8\} - \tfrac{0}{2}\{2x_1 + x_2 - x_3 + 2x_4 = -2\}$$

$$\rightarrow 3x_2 - 6x_3 + x_4 = 8$$

$$-x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \quad \rightarrow \{-x_1 + 3x_2 - 5x_3 - 6x_4 = 3\}$$

$$-\tfrac{(-1)}{2}\{2x_1 + x_2 - x_3 + 2x_4 = -2\}$$

$$\rightarrow \tfrac{7}{2}x_2 - \tfrac{11}{2}x_3 - 5x_4 = 2$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array}$$

$$3x_2 - 6x_3 + x_4 = 8 \quad \rightarrow \{3x_2 - 6x_3 + x_4 = 8\} - \tfrac{0}{2}\{2x_1 + x_2 - x_3 + 2x_4 = -2\}$$
$$\rightarrow 3x_2 - 6x_3 + x_4 = 8$$

$$-x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \quad \rightarrow \{-x_1 + 3x_2 - 5x_3 - 6x_4 = 3\}$$
$$-\tfrac{(-1)}{2}\{2x_1 + x_2 - x_3 + 2x_4 = -2\}$$
$$\rightarrow \tfrac{7}{2}x_2 - \tfrac{11}{2}x_3 - 5x_4 = 2$$

$$x_1 + 4x_2 - 3x_3 + x_4 = 4 \quad \rightarrow \{x_1 + 4x_2 - 3x_3 + x_4 = 4\}$$
$$-\tfrac{1}{2}\{2x_1 + x_2 - x_3 + 2x_4 = -2\}$$
$$\rightarrow \tfrac{7}{2}x_2 - \tfrac{5}{2}x_3 = 5$$

Ejemplo:

$$\begin{aligned} 3x_2 - 6x_3 + x_4 &= 8 \\ 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \rightarrow \begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \rightarrow$$

$$\begin{aligned} 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ 3x_2 - 6x_3 + x_4 &= 8 \\ \tfrac{7}{2}x_2 - \tfrac{11}{2}x_3 - 5x_4 &= 2 \\ \tfrac{7}{2}x_2 - \tfrac{5}{2}x_3 &= 5 \end{aligned}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array}$$

$$3x_2 - 6x_3 + x_4 = 8 \quad \rightarrow \left\{3x_2 - 6x_3 + x_4 = 8\right\} - \frac{3}{\left(\frac{7}{2}\right)}\left\{\frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2\right\}$$

$$\rightarrow -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array}$$

$$3x_2 - 6x_3 + x_4 = 8 \quad \rightarrow \left\{3x_2 - 6x_3 + x_4 = 8\right\} - \frac{3}{\left(\frac{7}{2}\right)}\left\{\frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2\right\}$$

$$\rightarrow -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7}$$

$$\frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \quad \rightarrow \left\{\frac{7}{2}x_2 - \frac{5}{2}x_3 = 5\right\} - \frac{\left(\frac{7}{2}\right)}{\left(\frac{7}{2}\right)}\left\{\frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2\right\}$$

$$\rightarrow 3x_3 + 5x_4 = 3$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \\ 3x_3 + 5x_4 = 3 \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \\ 3x_3 + 5x_4 = 3 \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \\ 3x_3 + 5x_4 = 3 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_3 + 5x_4 = 3 \\ -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \\ 3x_3 + 5x_4 = 3 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_3 + 5x_4 = 3 \\ -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \end{array}$$

$$-\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \quad \rightarrow \left\{ -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \right\} - \frac{\frac{-9}{7}}{3}\left\{ 3x_3 + 5x_4 = 3 \right\}$$

$$\rightarrow \frac{52}{7}x_4 = \frac{53}{7}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \\ 3x_3 + 5x_4 = 3 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_3 + 5x_4 = 3 \\ -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_3 + 5x_4 = 3 \\ \frac{52}{7}x_4 = \frac{53}{7} \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_2 - 6x_3 + x_4 = 8 \\ \frac{7}{2}x_2 - \frac{5}{2}x_3 = 5 \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \\ 3x_3 + 5x_4 = 3 \end{array} \rightarrow \begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_3 + 5x_4 = 3 \\ -\frac{9}{7}x_3 + \frac{37}{7}x_4 = \frac{44}{7} \end{array} \rightarrow$$

$$\begin{array}{r} 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ \frac{7}{2}x_2 - \frac{11}{2}x_3 - 5x_4 = 2 \\ 3x_3 + 5x_4 = 3 \\ \frac{52}{7}x_4 = \frac{53}{7} \end{array} \rightarrow \begin{array}{l} x_4 = \frac{53}{52}, \quad x_3 = -\frac{109}{156}, \\ x_2 = \frac{145}{156}, \quad x_1 = -\frac{17}{6} \end{array}$$

Método de eliminación de Gauss con elección de pivote (II):

Para minimizar aún más los errores de redondeo podemos aplicar el método de Gauss con "**pivote total**". En ese caso, en cada etapa del proceso, pasaremos de un sistema de la forma

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n \end{aligned}$$

a otro equivalente de la forma

$$\begin{aligned} a'_{11}x'_1 + a'_{12}x'_2 + \cdots + a'_{1n}x'_n &= b'_1 \\ a'_{22}x'_2 + \cdots + a'_{2n}x'_n &= b'_2 \\ \cdots\cdots\cdots\cdots\cdots & \\ a'_{n2}x'_2 + \cdots + a'_{nn}x'_n &= b'_n \end{aligned}$$

reordenando las ecuaciones (e incógnitas) originales para que $a'_{11} = a_{IJ}$ y $x'_1 = x_J$, siendo

$$|a_{IJ}| = \max_{1 \leq i,j \leq n} |a_{ij}|.$$

Ejemplo:

Ejemplo:

$$\begin{aligned} 3x_2 - 6x_3 + x_4 &= 8 \\ 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned}$$

Ejemplo:

$$
\begin{aligned}
3x_2 - 6x_3 + x_4 &= 8 \\
2x_1 + x_2 - x_3 + 2x_4 &= -2 \\
-x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\
x_1 + 4x_2 - 3x_3 + x_4 &= 4
\end{aligned}
$$

Ejemplo:

$$\begin{aligned} 3x_2 - 6x_3 + x_4 &= 8 \\ 2x_1 + x_2 - x_3 + 2x_4 &= -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 &= 3 \\ x_1 + 4x_2 - 3x_3 + x_4 &= 4 \end{aligned} \quad \rightarrow \quad \begin{aligned} -6x_3 + 3x_2 + x_4 &= 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 &= -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 &= 3 \\ -3x_3 + x_1 + 4x_2 + x_4 &= 4 \end{aligned}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array}$$

$$-x_3 + 2x_1 + x_2 + 2x_4 = -2 \quad \rightarrow \{ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \}$$

$$-\frac{(-1)}{-6}\{ -6x_3 + 3x_2 + x_4 = 8 \}$$

$$\rightarrow 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array}$$

$$-x_3 + 2x_1 + x_2 + 2x_4 = -2 \quad \rightarrow \{ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \}$$

$$-\frac{(-1)}{-6}\{ -6x_3 + 3x_2 + x_4 = 8 \}$$

$$\rightarrow 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3}$$

$$-5x_3 - x_1 + 3x_2 - 6x_4 = 3 \quad \rightarrow \{ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \}$$

$$-\frac{(-5)}{-6}\{ -6x_3 + 3x_2 + x_4 = 8 \}$$

$$\rightarrow -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array}$$

$$-x_3 + 2x_1 + x_2 + 2x_4 = -2 \quad \rightarrow \{ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \}$$

$$-\frac{(-1)}{-6}\{ -6x_3 + 3x_2 + x_4 = 8 \}$$

$$\rightarrow 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3}$$

$$-5x_3 - x_1 + 3x_2 - 6x_4 = 3 \quad \rightarrow \{ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \}$$

$$-\frac{(-5)}{-6}\{ -6x_3 + 3x_2 + x_4 = 8 \}$$

$$\rightarrow -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3}$$

$$-3x_3 + x_1 + 4x_2 + x_4 = 4 \quad \rightarrow \{ -3x_3 + x_1 + 4x_2 + x_4 = 4 \}$$

$$-\frac{(-3)}{-6}\{ -6x_3 + 3x_2 + x_4 = 8 \}$$

$$\rightarrow x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3} \\ -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3} \\ x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0 \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3} \\ -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3} \\ x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0 \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3} \\ -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3} \\ x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \\ \frac{1}{2}x_4 + x_1 + \frac{5}{2}x_2 = 0 \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3} \\ -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3} \\ x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \\ \frac{1}{2}x_4 + x_1 + \frac{5}{2}x_2 = 0 \end{array}$$

$$\frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \rightarrow \left\{ \frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \right\}$$

$$-\frac{\frac{11}{6}}{-\frac{41}{6}} \left\{ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \right\}$$

$$\rightarrow \frac{71}{41}x_1 + \frac{26}{41}x_2 = -\frac{177}{41}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3} \\ -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3} \\ x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \\ \frac{1}{2}x_4 + x_1 + \frac{5}{2}x_2 = 0 \end{array}$$

$$\frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \rightarrow \left\{ \frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \right\}$$

$$-\frac{\frac{11}{6}}{-\frac{41}{6}} \left\{ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \right\}$$

$$\rightarrow \frac{71}{41}x_1 + \frac{26}{41}x_2 = -\frac{177}{41}$$

$$\frac{1}{2}x_4 + x_1 + \frac{5}{2}x_2 = 0 \rightarrow \left\{ \frac{1}{2}x_4 + x_1 + \frac{5}{2}x_2 = 0 \right\}$$

$$-\frac{\frac{1}{2}}{-\frac{41}{6}} \left\{ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \right\}$$

$$\rightarrow \frac{38}{41}x_1 + \frac{104}{41}x_2 = -\frac{11}{41}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3} \\ -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3} \\ x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \\ \frac{1}{2}x_4 + x_1 + \frac{5}{2}x_2 = 0 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{71}{41}x_1 + \frac{26}{41}x_2 = -\frac{177}{41} \\ \frac{38}{41}x_1 + \frac{104}{41}x_2 = -\frac{11}{41} \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3} \\ -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3} \\ x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \\ \frac{1}{2}x_4 + x_1 + \frac{5}{2}x_2 = 0 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{71}{41}x_1 + \frac{26}{41}x_2 = -\frac{177}{41} \\ \frac{38}{41}x_1 + \frac{104}{41}x_2 = -\frac{11}{41} \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3} \\ -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3} \\ x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \\ \frac{1}{2}x_4 + x_1 + \frac{5}{2}x_2 = 0 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{71}{41}x_1 + \frac{26}{41}x_2 = -\frac{177}{41} \\ \frac{38}{41}x_1 + \frac{104}{41}x_2 = -\frac{11}{41} \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{104}{41}x_2 + \frac{38}{41}x_1 = -\frac{11}{41} \\ \frac{26}{41}x_2 + \frac{71}{41}x_1 = -\frac{177}{41} \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3} \\ -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3} \\ x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \\ \frac{1}{2}x_4 + x_1 + \frac{5}{2}x_2 = 0 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{71}{41}x_1 + \frac{26}{41}x_2 = -\frac{177}{41} \\ \frac{38}{41}x_1 + \frac{104}{41}x_2 = -\frac{11}{41} \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{104}{41}x_2 + \frac{38}{41}x_1 = -\frac{11}{41} \\ \frac{26}{41}x_2 + \frac{71}{41}x_1 = -\frac{177}{41} \end{array}$$

$$\frac{26}{41}x_2 + \frac{71}{41}x_1 = -\frac{177}{41} \quad \rightarrow \left\{ \frac{26}{41}x_2 + \frac{71}{41}x_1 = -\frac{177}{41} \right\}$$

$$-\frac{\frac{26}{41}}{\frac{104}{41}} \left\{ \frac{104}{41}x_2 + \frac{38}{41}x_1 = -\frac{11}{41} \right\} \rightarrow \frac{3}{2}x_1 = -\frac{17}{4}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3} \\ -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3} \\ x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \\ \frac{1}{2}x_4 + x_1 + \frac{5}{2}x_2 = 0 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{71}{41}x_1 + \frac{26}{41}x_2 = -\frac{177}{41} \\ \frac{38}{41}x_1 + \frac{104}{41}x_2 = -\frac{11}{41} \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{104}{41}x_2 + \frac{38}{41}x_1 = -\frac{11}{41} \\ \frac{26}{41}x_2 + \frac{71}{41}x_1 = -\frac{177}{41} \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{104}{41}x_2 + \frac{38}{41}x_1 = -\frac{11}{41} \\ \frac{3}{2}x_1 = -\frac{17}{4} \end{array}$$

Ejemplo:

$$\begin{array}{r} 3x_2 - 6x_3 + x_4 = 8 \\ 2x_1 + x_2 - x_3 + 2x_4 = -2 \\ -x_1 + 3x_2 - 5x_3 - 6x_4 = 3 \\ x_1 + 4x_2 - 3x_3 + x_4 = 4 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -x_3 + 2x_1 + x_2 + 2x_4 = -2 \\ -5x_3 - x_1 + 3x_2 - 6x_4 = 3 \\ -3x_3 + x_1 + 4x_2 + x_4 = 4 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ 2x_1 + \frac{1}{2}x_2 + \frac{11}{6}x_4 = -\frac{10}{3} \\ -x_1 + \frac{1}{2}x_2 - \frac{41}{6}x_4 = -\frac{11}{3} \\ x_1 + \frac{5}{2}x_2 + \frac{1}{2}x_4 = 0 \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{11}{6}x_4 + 2x_1 + \frac{1}{2}x_2 = -\frac{10}{3} \\ \frac{1}{2}x_4 + x_1 + \frac{5}{2}x_2 = 0 \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{71}{41}x_1 + \frac{26}{41}x_2 = -\frac{177}{41} \\ \frac{38}{41}x_1 + \frac{104}{41}x_2 = -\frac{11}{41} \end{array} \rightarrow \begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{104}{41}x_2 + \frac{38}{41}x_1 = -\frac{11}{41} \\ \frac{26}{41}x_2 + \frac{71}{41}x_1 = -\frac{177}{41} \end{array} \rightarrow$$

$$\begin{array}{r} -6x_3 + 3x_2 + x_4 = 8 \\ -\frac{41}{6}x_4 - x_1 + \frac{1}{2}x_2 = -\frac{11}{3} \\ \frac{104}{41}x_2 + \frac{38}{41}x_1 = -\frac{11}{41} \\ \frac{3}{2}x_1 = -\frac{17}{4} \end{array} \rightarrow \begin{array}{l} x_1 = -\frac{17}{6} \rightarrow x_2 = \frac{145}{156} \rightarrow \\ x_4 = \frac{53}{52} \rightarrow x_3 = -\frac{109}{156} \end{array}$$

Repaso de algunos conceptos.

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix} =$$

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix} = 1 \cdot (-2) - 3 \cdot 4 = -14$$

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix} = 1 \cdot (-2) - 3 \cdot 4 = -14 \qquad \det(27) =$$

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det \begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix} = 1 \cdot (-2) - 3 \cdot 4 = -14 \qquad \det(27) = 27$$

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det\begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix} = 1\cdot(-2) - 3\cdot 4 = -14 \qquad \det(27) = 27$$

$$\det\begin{pmatrix} 1 & 3 & 5 \\ 4 & -2 & 2 \\ -1 & 3 & 6 \end{pmatrix} =$$

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det\begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix} = 1\cdot(-2) - 3\cdot 4 = -14 \qquad \det(27) = 27$$

$$\det\begin{pmatrix} 1 & 3 & 5 \\ 4 & -2 & 2 \\ -1 & 3 & 6 \end{pmatrix} = \begin{array}{l} 1\cdot(-2)\cdot 6 + 3\cdot 2\cdot(-1) + 5\cdot 4\cdot 3 \\ \quad - 5\cdot(-2)\cdot(-1) - 3\cdot 4\cdot 6 - 1\cdot 2\cdot 3 = -46 \end{array}$$

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det\begin{pmatrix}1 & 3\\ 4 & -2\end{pmatrix} = 1\cdot(-2) - 3\cdot 4 = -14 \qquad \det(27) = 27$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = \begin{array}{l}1\cdot(-2)\cdot 6 + 3\cdot 2\cdot(-1) + 5\cdot 4\cdot 3\\ -5\cdot(-2)\cdot(-1) - 3\cdot 4\cdot 6 - 1\cdot 2\cdot 3 = -46\end{array}$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} =$$

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det\begin{pmatrix}1 & 3\\ 4 & -2\end{pmatrix} = 1\cdot(-2) - 3\cdot 4 = -14 \qquad \det(27) = 27$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = \begin{array}{l}1\cdot(-2)\cdot 6 + 3\cdot 2\cdot(-1) + 5\cdot 4\cdot 3\\ -5\cdot(-2)\cdot(-1) - 3\cdot 4\cdot 6 - 1\cdot 2\cdot 3 = -46\end{array}$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = 1\cdot\det\begin{pmatrix}-2 & 2\\ 3 & 6\end{pmatrix} - 4\cdot\det\begin{pmatrix}3 & 5\\ 3 & 6\end{pmatrix} + (-1)\cdot\det\begin{pmatrix}3 & 5\\ -2 & 2\end{pmatrix} =$$
$$1\cdot(-18) - 4\cdot 3 + (-1)\cdot 16 = -46$$

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det\begin{pmatrix}1 & 3\\ 4 & -2\end{pmatrix} = 1\cdot(-2) - 3\cdot 4 = -14 \qquad \det(27) = 27$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = \begin{array}{l}1\cdot(-2)\cdot 6 + 3\cdot 2\cdot(-1) + 5\cdot 4\cdot 3\\ -5\cdot(-2)\cdot(-1) - 3\cdot 4\cdot 6 - 1\cdot 2\cdot 3 = -46\end{array}$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = \begin{array}{l}1\cdot\det\begin{pmatrix}-2 & 2\\ 3 & 6\end{pmatrix} - 4\cdot\det\begin{pmatrix}3 & 5\\ 3 & 6\end{pmatrix} + (-1)\cdot\det\begin{pmatrix}3 & 5\\ -2 & 2\end{pmatrix} =\\ 1\cdot(-18) - 4\cdot 3 + (-1)\cdot 16 = -46\end{array}$$

- El determinante de una matriz triangular superior (o inferior) es igual al producto de los elementos de su diagonal.

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det\begin{pmatrix}1 & 3\\ 4 & -2\end{pmatrix} = 1\cdot(-2) - 3\cdot 4 = -14 \qquad \det(27) = 27$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = \begin{array}{l}1\cdot(-2)\cdot 6 + 3\cdot 2\cdot(-1) + 5\cdot 4\cdot 3\\ -5\cdot(-2)\cdot(-1) - 3\cdot 4\cdot 6 - 1\cdot 2\cdot 3 = -46\end{array}$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = 1\cdot\det\begin{pmatrix}-2 & 2\\ 3 & 6\end{pmatrix} - 4\cdot\det\begin{pmatrix}3 & 5\\ 3 & 6\end{pmatrix} + (-1)\cdot\det\begin{pmatrix}3 & 5\\ -2 & 2\end{pmatrix} =$$
$$1\cdot(-18) - 4\cdot 3 + (-1)\cdot 16 = -46$$

- El determinante de una matriz triangular superior (o inferior) es igual al producto de los elementos de su diagonal.

$$\det\begin{pmatrix}1 & 3 & 5\\ 0 & -2 & 2\\ 0 & 0 & 6\end{pmatrix} = 1\cdot(-2)\cdot 6 = -12$$

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det\begin{pmatrix}1 & 3\\ 4 & -2\end{pmatrix} = 1\cdot(-2) - 3\cdot 4 = -14 \qquad \det(27) = 27$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = \begin{array}{l}1\cdot(-2)\cdot 6 + 3\cdot 2\cdot(-1) + 5\cdot 4\cdot 3\\ -5\cdot(-2)\cdot(-1) - 3\cdot 4\cdot 6 - 1\cdot 2\cdot 3 = -46\end{array}$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = 1\cdot\det\begin{pmatrix}-2 & 2\\ 3 & 6\end{pmatrix} - 4\cdot\det\begin{pmatrix}3 & 5\\ 3 & 6\end{pmatrix} + (-1)\cdot\det\begin{pmatrix}3 & 5\\ -2 & 2\end{pmatrix} =$$
$$1\cdot(-18) - 4\cdot 3 + (-1)\cdot 16 = -46$$

NO es cierto que el determinante de la suma de matrices sea la suma de determinantes.

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det\begin{pmatrix}1 & 3\\ 4 & -2\end{pmatrix} = 1\cdot(-2) - 3\cdot 4 = -14 \qquad \det(27) = 27$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = \begin{array}{l}1\cdot(-2)\cdot 6 + 3\cdot 2\cdot(-1) + 5\cdot 4\cdot 3\\ -5\cdot(-2)\cdot(-1) - 3\cdot 4\cdot 6 - 1\cdot 2\cdot 3 = -46\end{array}$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = 1\cdot\det\begin{pmatrix}-2 & 2\\ 3 & 6\end{pmatrix} - 4\cdot\det\begin{pmatrix}3 & 5\\ 3 & 6\end{pmatrix} + (-1)\cdot\det\begin{pmatrix}3 & 5\\ -2 & 2\end{pmatrix} =$$
$$1\cdot(-18) - 4\cdot 3 + (-1)\cdot 16 = -46$$

NO es cierto que el determinante de la suma de matrices sea la suma de determinantes.

$$\det\left(\begin{pmatrix}-2 & 2\\ 3 & 6\end{pmatrix} + \begin{pmatrix}3 & 5\\ 3 & 6\end{pmatrix}\right) = \det\begin{pmatrix}1 & 7\\ 6 & 12\end{pmatrix} = -30$$

$$\det\begin{pmatrix}-2 & 2\\ 3 & 6\end{pmatrix} + \det\begin{pmatrix}3 & 5\\ 3 & 6\end{pmatrix} = -18 + 3 = -15 \neq -30$$

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det\begin{pmatrix}1 & 3\\ 4 & -2\end{pmatrix} = 1\cdot(-2) - 3\cdot 4 = -14 \qquad \det(27) = 27$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = \begin{array}{l}1\cdot(-2)\cdot 6 + 3\cdot 2\cdot(-1) + 5\cdot 4\cdot 3\\ -5\cdot(-2)\cdot(-1) - 3\cdot 4\cdot 6 - 1\cdot 2\cdot 3 = -46\end{array}$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = 1\cdot\det\begin{pmatrix}-2 & 2\\ 3 & 6\end{pmatrix} - 4\cdot\det\begin{pmatrix}3 & 5\\ 3 & 6\end{pmatrix} + (-1)\cdot\det\begin{pmatrix}3 & 5\\ -2 & 2\end{pmatrix} =$$
$$1\cdot(-18) - 4\cdot 3 + (-1)\cdot 16 = -46$$

SI es cierto que el determinante del producto de matrices es el producto de determinantes. Ejemplo:

Repaso de algunos conceptos.

Antes de continuar, debemos recordar como se calcula el determinante de una matriz:

$$\det\begin{pmatrix}1 & 3\\ 4 & -2\end{pmatrix} = 1\cdot(-2) - 3\cdot 4 = -14 \qquad \det(27) = 27$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = \begin{array}{l}1\cdot(-2)\cdot 6 + 3\cdot 2\cdot(-1) + 5\cdot 4\cdot 3\\ -5\cdot(-2)\cdot(-1) - 3\cdot 4\cdot 6 - 1\cdot 2\cdot 3 = -46\end{array}$$

$$\det\begin{pmatrix}1 & 3 & 5\\ 4 & -2 & 2\\ -1 & 3 & 6\end{pmatrix} = 1\cdot\det\begin{pmatrix}-2 & 2\\ 3 & 6\end{pmatrix} - 4\cdot\det\begin{pmatrix}3 & 5\\ 3 & 6\end{pmatrix} + (-1)\cdot\det\begin{pmatrix}3 & 5\\ -2 & 2\end{pmatrix} =$$
$$1\cdot(-18) - 4\cdot 3 + (-1)\cdot 16 = -46$$

SI es cierto que el determinante del producto de matrices es el producto de determinantes. Ejemplo:

$$\det\left(\begin{pmatrix}-2 & 2\\ 3 & 6\end{pmatrix}\begin{pmatrix}3 & 5\\ 3 & 6\end{pmatrix}\right) = \det\begin{pmatrix}-6+6 & -10+12\\ 9+18 & 15+36\end{pmatrix} =$$
$$\det\begin{pmatrix}0 & 2\\ 27 & 51\end{pmatrix} = -54 = (-18)3 = \det\begin{pmatrix}-2 & 2\\ 3 & 6\end{pmatrix}\det\begin{pmatrix}3 & 5\\ 3 & 6\end{pmatrix}$$

Recordemos también que los **menores principales** de la matriz

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

son

Recordemos también que los **menores principales** de la matriz

$$\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

son

$$\det(a_{11}),\ \det\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix},\ \det\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix},\ \ldots,\ \det\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$