

Sistemas lineales

- Jacobi: $Dx_{k+1} = -(L+U)x_k + b$. Gauss-Seidel: $(D+L)x_{k+1} = -Ux_k + b$
- Relajación sucesiva: $(\mathbf{D} + \omega\mathbf{L})\mathbf{x} = ((1-\omega)\mathbf{D} - \omega\mathbf{U})\mathbf{x} + \omega\mathbf{b}$.
- Reflexión de Householder: $P = I - 2vv^t$.

Resolución de ecuaciones

- Newton-Raphson: $x_{n+1} = x_n - f(x_n)/f'(x_n)$, $E_{n+1} = \frac{-f''(\xi)}{2f'(x_n)}E_n^2$.
- Punto fijo: $x_n = F(x_{n-1})$, $E_{n+1} \leq K^n \frac{|x_1 - x_0|}{1-K}$
- Fórmula de extrapolación de Aitken: $c - x_n \approx \frac{\lambda_n}{1-\lambda_n}(x_n - x_{n-1})$,
 $\lambda_n := \frac{x_n - x_{n-1}}{x_{n-1} - x_{n-2}}$.
- Sistemas no lineales (Newton): $\mathbf{J}(\mathbf{x}_n)(\mathbf{x}_{n+1} - \mathbf{x}_n) = -F(\mathbf{x}_n)$

Interpolación y aproximación

- Error de interpolación: $f(x) - P_n(x) = \frac{f^{(n+1)}}{(n+1)!}(x-x_0)(x-x_1)\dots(x-x_n)$
- Diferencias divididas: $f[x_k] = f(x_k)$, $f[x_i, x_{i+1}, \dots, x_j] = \frac{f[x_{i+1}, x_{i+2}, \dots, x_j] - f[x_i, x_{i+1}, \dots, x_{j-1}]}{x_j - x_i}$
- Momentos: $h_i := x_{i+1} - x_i$, $b_i := \frac{y_{i+1} - y_i}{h_i}$, $u_i = 2(h_{i-1} + h_i)$, $v_i = 6(b_i - b_{i-1})$.

Sistema:

$$\begin{cases} z_0 & = 0, \\ h_{i-1}z_{i-1} + u_i z_i + h_i z_{i+1} & = v_i, \quad (1 \leq i \leq n-1) \\ z_n & = 0, \end{cases}$$

Polinomios ($0 \leq i \leq n-1$):

$$S_i(x) = \frac{z_{i+1}}{6h_i}(x-x_i)^3 + \frac{z_i}{6h_i}(x_{i+1}-x)^3 + \left(\frac{y_{i+1}}{h_i} - \frac{h_i}{6}z_{i+1}\right)(x-x_i) + \left(\frac{y_i}{h_i} - \frac{h_i}{6}z_i\right)(x_{i+1}-x).$$

Integración

- Trapecio: $\int_a^b f(x) dx = \frac{h}{2}(f(a) + f(b)) - \frac{h^3}{12}f''(\xi)$.
- Simpson: $\int_a^b f(x) dx = \frac{h}{3}(f(a) + 4f(x_1) + f(b)) - \frac{h^5}{90}f^{(4)}(\xi)$.
- Error métodos compuestos:
 Trapecio: $-\frac{f''(\xi)}{12}(b-a)h^2$, Simpson: $-\frac{f^{(4)}(\xi)}{180}h^4(b-a)$.
- Polinomios de Legendre: $P_0(x) = 1$, $P_1(x) = x$,
 $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$.